MATH-944 Semigroup Theory of Operators

Credit Hours: 3-0

Prerequisite: None

Course Objectives: PhD/M.Phil and graduate students of functional analysis, applied mathematics, physics and engineering will find this an invaluable introduction to the subject. Main aim is to introduce students to the solutions of problems involving evolution equations via the theory of semigroup of operators. This course will enable the students to proceed to advanced textbooks and to many research papers devoted to the use of semigroups in the study of evolution systems.

Core Contents: Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, Well-posedness for evolution equations, Semilinear problems.

Course Contents: Spaces and operators, spectral theory, fixed point theorem, uniformly continuous operator semigroups, semigroups on Banach spaces, semigroups on Hilbert spaces, strongly continuous semigroups, generators of semigroups, Hille-Yosida theorems, dissipative and m- dissipative operators, construction of semigroups, perturbation of generators, abstract Cauchy problems, inhomogeneous Cauchy problems, semilinear ACP, mild solutions, strong solutions.

Learning Outcomes: Students are expected to understand Spectral Theory, Cauchy's Functional Equation, Semigroups on Banach and Hilbert spaces, Strongly continuous semigroups, and applications of semigroup operator theory in differential equations and functional equations.

Text Books:

1. Bellani-Morante and A. C. McBride, Applied Nonlinear Semigroups, John Wiley & Sons(Referred as BM)
2. K-J Engel and R. Nagel, One Parameter Semigroups for Linear Evolution Equations Springer(Referred as EN)

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	BM 1.7-1.11	Spaces and Operators, spectral Theory, Fixed Point Theorem
2	EN Chapter 1 1.1-1.4, 2.1-2.11	Cauchy's Functional Equation, Finite Dimensional Systems
3	Chapter 1 Section 3	Uniformly continuous operator semigroups, semigroups on Banach spaces, Semigroups on Hilbert spaces
4	Chapter 1 4.1 – 4.8	Multiplication Semigroups, Translation semigroups
5	Chapter 1 Section 5	Strongly continuous semigroups and its basic properties
6	Chapter 2 1.1-1.7	Construction and examples of strongly continuous semigroups
7	1.8-1.14	Generator of Semigroups and their resolvents
8	Chapter 2 2.1-2.11	Standard construction of similar semigroups, rescaled semigroups, subspace semigroups, quotient semigroups, adjoint semigroups, Product semigroups
9	**Mid Semester Exam**	
10	3.1-3.11	Hille-Yosida Generation Theorems
11	3.13-3.23	Dissipative Operators and Contractive Semigroups
13	4.1-4.15	Special classes of semigroups
14	Chapter 2 6.1-6.11	Well-posedness for evolution equations, abstract Cauchy problems, Inhomogeneous abstract Cauchy

	BM 2.5	problem and its strong solutions
15	Chapter 3 1.-1.15 BM 2.4	Perturbation of Generators, the Trotter-Kato theorems
16	BM 3.1- 3.2	Semilinear problems
17	BM 3.3- 3.4	strong solutions, mild solutions
17	Review	
18	**End Semester Exam**	